

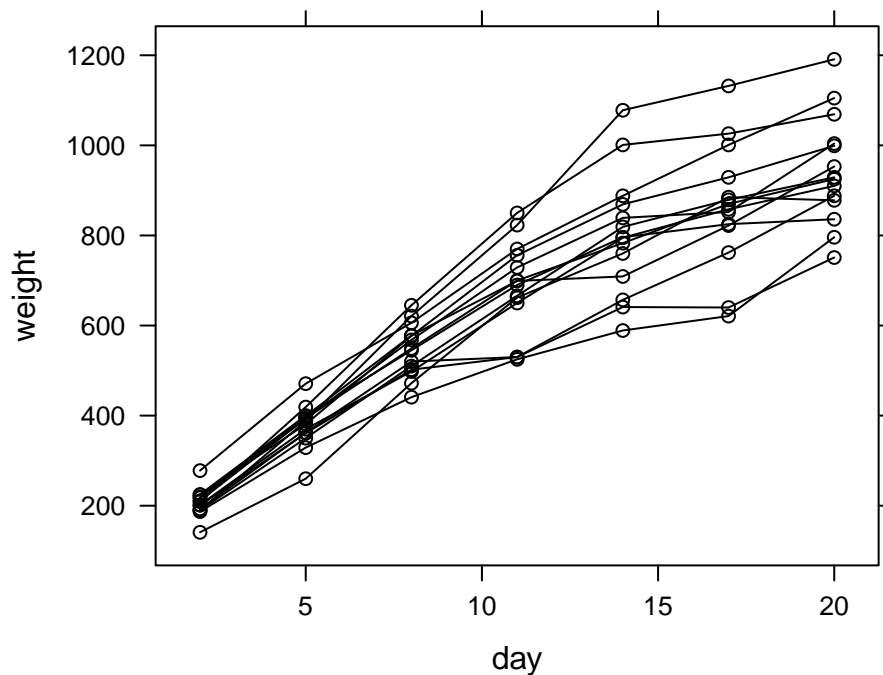
## Today...

- Why parameterize the covariance matrix
- Three models:
  - AR(1)
  - Compound symmetry = Random intercept
  - Random intercept and slope
- Fitting in R

## Small Mice data

```
> sm <- read.delim("http://rem.ph.ucla.edu/rob/mld/data/tabdelimiteddata/smallmice.txt")
> sm$dayF <- factor(sm$day)

> mtheme <- standard.theme("pdf")
> mtheme$superpose.symbol$col <- mtheme$superpose.line$col <- "black"
> plot(xyplot(weight ~ day, group = id, data = sm, type = "b",
+   par.settings = mtheme))
```



**Independent errors, fit using lm**

```
> m1 <- lm(weight ~ dayF, data = sm)
> summary(m1)
```

Call:

```
lm(formula = weight ~ dayF, data = sm)
```

Residuals:

Min	1Q	Median	3Q	Max
-243.429	-38.393	1.286	36.179	276.286

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	206.29	26.41	7.811	9.42e-12	***
dayF5	170.64	37.35	4.569	1.54e-05	***
dayF8	338.86	37.35	9.073	2.23e-14	***
dayF11	478.00	37.35	12.798	< 2e-16	***
dayF14	595.43	37.35	15.942	< 2e-16	***
dayF17	658.14	37.35	17.621	< 2e-16	***
dayF20	739.00	37.35	19.786	< 2e-16	***

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 98.82 on 91 degrees of freedom

Multiple R-squared: 0.8733, Adjusted R-squared: 0.865

F-statistic: 104.6 on 6 and 91 DF, p-value: < 2.2e-16

nlme vs lme4

```
> library(nlme)
```

Independent errors, fit using gls

```
> m2 <- gls(weight ~ dayF, data = sm)
> summary(m2)
```

Generalized least squares fit by REML

Model: weight ~ dayF

Data: sm

	AIC	BIC	logLik
	1128.695	1148.781	-556.3473

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	206.2857	26.40985	7.810940	0
dayF5	170.6429	37.34916	4.568854	0
dayF8	338.8571	37.34916	9.072684	0
dayF11	478.0000	37.34916	12.798145	0
dayF14	595.4286	37.34916	15.942220	0
dayF17	658.1429	37.34916	17.621355	0
dayF20	739.0000	37.34916	19.786253	0

Correlation:

	(Intr)	dayF5	dayF8	dayF11	dayF14	dayF17
dayF5	-0.707					
dayF8	-0.707	0.500				
dayF11	-0.707	0.500	0.500			
dayF14	-0.707	0.500	0.500	0.500		
dayF17	-0.707	0.500	0.500	0.500	0.500	
dayF20	-0.707	0.500	0.500	0.500	0.500	0.500

Standardized residuals:

	Min	Q1	Med	Q3	Max
	-2.46343812	-0.38852641	0.01301112	0.36611837	2.79594444

Residual standard error: 98.8166

Degrees of freedom: 98 total; 91 residual

**AR(1) model**

```
> m3 <- gls(weight ~ dayF, data = sm, correlation = corAR1(form = ~1 |
+   id))
> summary(m3)
```

Generalized least squares fit by REML

```
Model: weight ~ dayF
Data: sm
      AIC      BIC    logLik
1005.800 1028.398 -493.9002
```

Correlation Structure: AR(1)

```
Formula: ~1 | id
Parameter estimate(s):
      Phi
0.8781881
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	206.2857	25.01899	8.245165	0
dayF5	170.6429	12.34894	13.818422	0
dayF8	338.8571	16.92385	20.022461	0
dayF11	478.0000	20.10035	23.780680	0
dayF14	595.4286	22.52344	26.435949	0
dayF17	658.1429	24.45416	26.913330	0
dayF20	739.0000	26.03186	28.388285	0

Correlation:

	(Intr)	dayF5	dayF8	dayF11	dayF14	dayF17
dayF5	-0.247					
dayF8	-0.338	0.685				
dayF11	-0.402	0.544	0.791			
dayF14	-0.450	0.460	0.665	0.838		
dayF17	-0.489	0.403	0.580	0.728	0.865	
dayF20	-0.520	0.361	0.518	0.648	0.766	0.882

Standardized residuals:

	Min	Q1	Med	Q3	Max
	-2.60038533	-0.41012533	0.01373443	0.38647159	2.95137630

Residual standard error: 93.6125

Degrees of freedom: 98 total; 91 residual

Using `~ 1 | id` uses the order that the times appear in the data frame as the correct order. To put them in a different order, use `~ idx | id` where `idx` is an index variable (that is, it counts up 1, 2, 3, etc.) specifying the order. In this case we might create a variable like this `sm$dayN <- as.numeric(sm$dayF)` and then use the form `~ dayN | id`.

## Compound symmetry model

```
> m4 <- gls(weight ~ dayF, data = sm, correlation = corCompSymm(form = ~1 |
+   id))
> summary(m4)
```

Generalized least squares fit by REML

```
Model: weight ~ dayF
Data: sm
      AIC      BIC    logLik
1075.728 1098.326 -528.8642
```

Correlation Structure: Compound symmetry

```
Formula: ~1 | id
Parameter estimate(s):
  Rho
0.618237
```

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	206.2857	26.40985	7.81094	0
dayF5	170.6429	23.07692	7.39453	0
dayF8	338.8571	23.07692	14.68381	0
dayF11	478.0000	23.07692	20.71334	0
dayF14	595.4286	23.07692	25.80191	0
dayF17	658.1429	23.07692	28.51953	0
dayF20	739.0000	23.07692	32.02334	0

Correlation:

(Intr)	dayF5	dayF8	dayF11	dayF14	dayF17
dayF5	-0.437				
dayF8	-0.437	0.500			
dayF11	-0.437	0.500	0.500		
dayF14	-0.437	0.500	0.500	0.500	
dayF17	-0.437	0.500	0.500	0.500	0.500
dayF20	-0.437	0.500	0.500	0.500	0.500

Standardized residuals:

Min	Q1	Med	Q3	Max
-2.46343809	-0.38852640	0.01301112	0.36611837	2.79594441

Residual standard error: 98.8166

Degrees of freedom: 98 total; 91 residual

## Random intercept model

```
> m5 <- lme(weight ~ dayF, random = ~1 | id, data = sm)
> summary(m5)
```

Linear mixed-effects model fit by REML

```
Data: sm
      AIC      BIC    logLik
1075.728 1098.326 -528.8642
```

Random effects:

```
Formula: ~1 | id
      (Intercept) Residual
StdDev:    77.69756 61.05578
```

Fixed effects: weight ~ dayF

	Value	Std.Error	DF	t-value	p-value
(Intercept)	206.2857	26.40985	78	7.81094	0
dayF5	170.6429	23.07692	78	7.39453	0
dayF8	338.8571	23.07692	78	14.68381	0
dayF11	478.0000	23.07692	78	20.71334	0
dayF14	595.4286	23.07692	78	25.80191	0
dayF17	658.1429	23.07692	78	28.51953	0
dayF20	739.0000	23.07692	78	32.02334	0

Correlation:

	(Intr)	dayF5	dayF8	dayF11	dayF14	dayF17
dayF5	-0.437					
dayF8	-0.437	0.500				
dayF11	-0.437	0.500	0.500			
dayF14	-0.437	0.500	0.500	0.500		
dayF17	-0.437	0.500	0.500	0.500	0.500	
dayF20	-0.437	0.500	0.500	0.500	0.500	0.500

Standardized Within-Group Residuals:

	Min	Q1	Med	Q3	Max
	-2.41454078	-0.41439164	0.06718481	0.42107767	2.37733003

Number of Observations: 98

Number of Groups: 14

What are random effects?

How is this model the same as the compound symmetry model?

What if intraclass correlation is negative?

## Random intercept and slope model

```
> m6 <- lme(weight ~ dayF, random = ~day | id, data = sm)
> summary(m6)
```

Linear mixed-effects model fit by REML

```
Data: sm
      AIC      BIC    logLik
1014.264 1041.884 -496.1321
```

Random effects:

```
Formula: ~day | id
Structure: General positive-definite, Log-Cholesky parametrization
      StdDev   Corr
(Intercept) 27.347843 (Intr)
day          7.454573 -0.246
Residual    37.334238
```

Fixed effects: weight ~ dayF

	Value	Std.Error	DF	t-value	p-value
(Intercept)	206.2857	12.43193	78	16.59322	0
dayF5	170.6429	15.32465	78	11.13519	0
dayF8	338.8571	18.49369	78	18.32285	0
dayF11	478.0000	22.81746	78	20.94887	0
dayF14	595.4286	27.76157	78	21.44795	0
dayF17	658.1429	33.04876	78	19.91430	0
dayF20	739.0000	38.53809	78	19.17583	0

Correlation:

	(Intr)	dayF5	dayF8	dayF11	dayF14	dayF17
dayF5	-0.454					
dayF8	-0.319	0.603				
dayF11	-0.213	0.591	0.744			
dayF14	-0.137	0.570	0.751	0.834		
dayF17	-0.083	0.549	0.747	0.843	0.887	
dayF20	-0.044	0.532	0.741	0.844	0.894	0.920

Standardized Within-Group Residuals:

	Min	Q1	Med	Q3	Max
	-2.2352561	-0.5058675	0.0666112	0.5078845	2.2546801

Number of Observations: 98

Number of Groups: 14

## Unstructured variance matrix model

```
> m7 <- gls(weight ~ dayF, data = sm, correlation = corSymm(form = ~1 |
+   id))
> summary(m7)
```

Generalized least squares fit by REML

Model: weight ~ dayF

Data: sm

	AIC	BIC	logLik
	1010.776	1083.591	-476.3879

Correlation Structure: General

Formula: ~1 | id

Parameter estimate(s):

Correlation:

	1	2	3	4	5	6
1	0.978					
2	0.916	0.947				
3	0.679	0.728	0.855			
4	-0.163	-0.047	0.144	0.527		
5	0.059	0.141	0.320	0.673	0.897	
6	0.483	0.557	0.674	0.812	0.639	0.801

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	206.2857	27.25667	7.568264	0
dayF5	170.6429	5.69988	29.937965	0
dayF8	338.8571	11.16965	30.337313	0
dayF11	478.0000	21.83097	21.895496	0
dayF14	595.4286	41.57860	14.320555	0
dayF17	658.1429	37.39972	17.597533	0
dayF20	739.0000	27.71969	26.659751	0

Correlation:

	(Intr)	dayF5	dayF8	dayF11	dayF14	dayF17
dayF5	-0.105					
dayF8	-0.205	0.613				
dayF11	-0.400	0.420	0.791			
dayF14	-0.763	0.434	0.626	0.828		
dayF17	-0.686	0.362	0.615	0.851	0.957	
dayF20	-0.508	0.451	0.659	0.798	0.850	0.902

Standardized residuals:

	Min	Q1	Med	Q3	Max
	-2.38690251	-0.37645543	0.01260688	0.35474358	2.70907831

Residual standard error: 101.9851

Degrees of freedom: 98 total; 91 residual